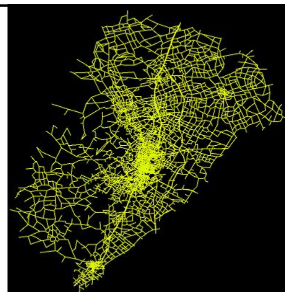

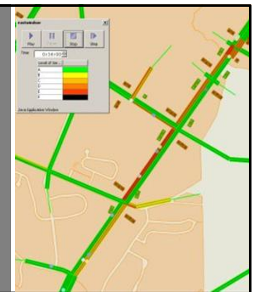

Dynamic Traffic Assignment

Basics Definitions and Properties

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TeQson Lab | Transportation EQuilibrium Simulation and Optimized Networks

Motivation

- DTA is emerging as a practical tool for numerous planning and operational applications
- Due to the recent advances in DTA, uncertainty remains on model capabilities, implications, etc
- The Primer attempts to address the core issues in DTA models

Motivation: What is a "good" DTA?

- A good model must provide sufficiently sound predictions of system behavior given the necessary resource constraints
- DTA provides a superior representation of traffic, dynamic conditions, and resulting behavior.
  - These benefits come at a computational and implementation cost

Motivation: What is a "good" DTA?

- Are the implementation costs worth the additional descriptive benefit?
- To answer, we must understand the underlying properties
- To understand a DTA approach fully the following must be precisely defined:
  - formulation assumption
  - solution method and
  - traffic model/simulation

Defining DTA

- Three things must be defined fully
  - Problem formulation
  - Solution method
  - Traffic model
- Why is there lack of consensus?
  - "Relatively" new
    - Well over a decade now of work, but there are lags between academic research and implementation.
  - Disconnect between research and practice
  - Solution methods more complex for dynamic than static
- **Somewhat similar evolution for static decades ago**

Defining DTA

- Static progressed from "primitive" approaches to more advanced solution methods (Sheffi, 1985) over the course of many years:
  - All-or-nothing
  - Capacity restrained
  - Modified capacity restrained
  - Incremental Assignment
  - MSA
  - **Frank-Wolfe**
  - Many newer advances past FW

Static to DTA: What should NOT Change

- **Formulation**
- There are two primary assumptions of equilibrium (whether static or dynamic)
  - **Users are "greedy"**
  - **Users are familiar with the system**
- While special cases and extensions may be considered (familiarity might be tuned with SUE for instance)
- These assumption should not change fundamentally simply because time is being considered

Formulation/Assumptions

- Behavioral assumptions are critical
- Equilibrium represents one of the ***simplest*** cases of behavior
  - *But equilibrium requires iteration*
- Without sound behavioral assumptions **transferability** and **consistency** are not achievable
  - Calibration alone (without sound behavior) does **not** imply transferability and consistency

Static to DTA: What should NOT Change

- **Solution Method**
- Static methods often employ ***at least*** Frank-Wolfe

- While perhaps <u>not ideal</u>, they provide some measure of
  - Convergence criteria
  - Efficiency
  - Consistency

- These can not be lost in an attempt to model temporal behavior

Why worry about formulation/method?

- Based on the assumptions of the problem formulation and the solution method
  - Substantially different results will be observed
  - Consistency will also alter dramatically and may be fully unachievable

Examples for Static

- For DTA, we first need to be sure on static methods
- First we need a static formulation
- This has been well established

$$\min \sum_a \int_0^{x_a} c_a(\omega) d\omega$$

$$s.t.$$

$$\sum_k h_k^{rs} = q_{rs} \qquad \forall r, s$$

$$h_k^{rs} \geq 0 \qquad \forall k, r, s$$

$$x_a = \sum_r \sum_s \sum_k h_k^{rs} \delta_{a,k}^{rs} \qquad \forall a$$

For Static: Solution Methods

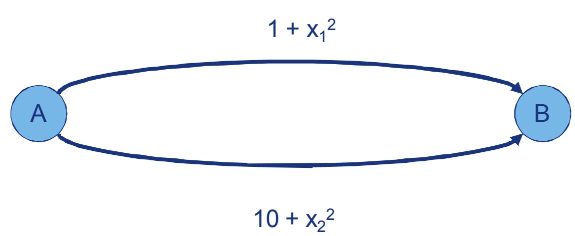
- Many solution methods work by “linearizing” the non-linear objective.
- The linearized version is simply the gradient of the function
- For UE, we are very lucky.
- Basically, by taking derivatives, the integrals disappear, leaving simply the cost functions!
- Therefore, to solve the linear problem we simply need to find the shortest path!
- This must be done many times though.

Static: Example of solution methods

- Quick examples of
  - Method of Successive Averages (MSA)
  - Frank Wolfe (FW)
- Closely see differences from solution method
  - Even when assumptions/formulation the same
- For DTA analogy

MSA

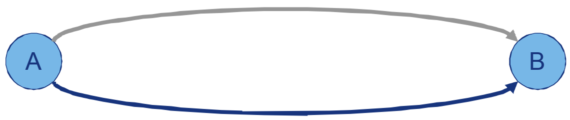
Demand A,B = 10



Begin by taking free-flow costs

MSA – Iteration 1

$1 + x_1^2 = 1$



$10 + x_2^2 = 10$

So All-or-nothing flow assignment is

$x_1' = 10$ and $x_2' = 0$

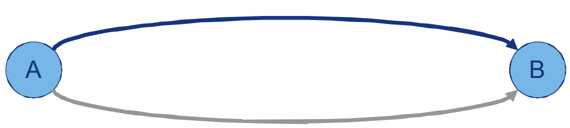
or $x = [\ 10 \quad 0\ ]$

Calculate new costs based on these flows

MSA - Iteration 2

$x = [\ 10 \quad 0\ ]$

$1 + x_1^2 = 101$



$10 + x_2^2 = 10$

So All-or-nothing flow assignment is $x_1' = 0$ and $x_2' = 10$

or $x_i = (½)\, x' + (1 – ½)\, x_{old}$ so

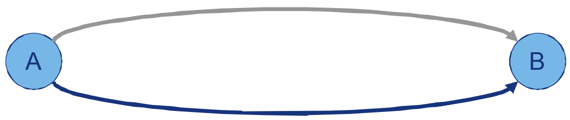
$x_1 = (½)\ 0 + (½)\ 10 \quad = 5$
$x_2 = (½)\ 10 + (½)\ 0 \quad = 5$

Calculate new costs based on these flows

MSA - Iteration 3

x = [5 5]

$1 + x_1^2 = 26$



$10 + x_2^2 = 35$

So All-or-nothing flow assignment is $x_1' = 10$ and $x_2' = 0$

or $x_i = (1/3)\,x' + (1 - 1/3)\,x_{old}$ so

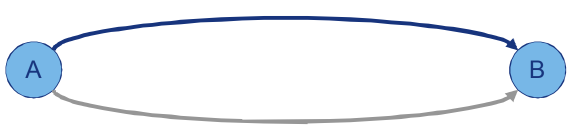
$x_1 = (1/3)\,10 + (2/3)\,5 = 6.66666$
$x_2 = (1/3)\,0 + (2/3)\,5 = 3.33333$

Calculate new costs based on these flows

MSA - Iteration 4

x = [6.666666 3.333333]

$1 + x_1^2 = 45.4$



$10 + x_2^2 = 21.9$

So All-or-nothing flow assignment is $x_1' = 0$ and $x_2' = 10$

or $x_i = (1/4)\,x' + (1 - 1/4)\,x_{old}$ so

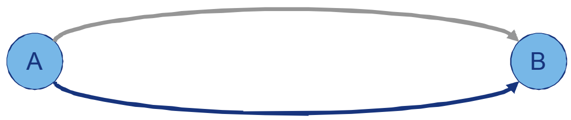
$x_1 = (1/4)\,0 + (3/4)\,6.66666 = 5$
$x_2 = (1/4)\,10 + (3/4)\,3.33333 = 5$

Calculate new costs based on these flows

MSA - Iteration 5

$x = [\,5 \;\; 5\,]$

$1 + x_1^2 = 26$



$10 + x_2^2 = 35$

So All-or-nothing flow assignment is $x_1' = 10$ and $x_2' = 0$

or $x_i = (1/5)\, x' + (1 - 1/5)\, x_{old}$ so

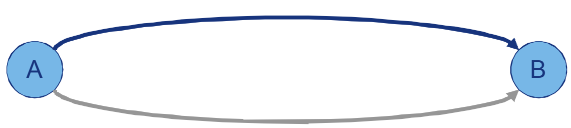
$x_1 = (1/5)\, 10 + (4/5)\, 5 = 6$
$x_2 = (1/5)\, 0 + (4/5)\, 5 = 4$

Calculate new costs based on these flows

MSA - Iteration 6

$x = [\,6 \;\; 4\,]$

$1 + x_1^2 = 37$



$10 + x_2^2 = 26$

So All-or-nothing flow assignment is $x_1' = 0$ and $x_2' = 10$

or $x_i = (1/6)\, x' + (1 - 1/6)\, x_{old}$ so

$x_1 = (1/6)\, 10 + (5/6)\, 6 = 6.66666$
$x_2 = (1/6)\, 0 + (5/6)\, 4 = 3.33333$

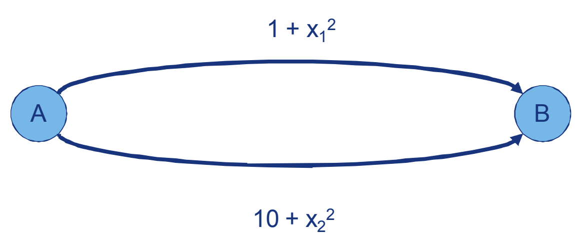
Calculate new costs based on these flows

MSA

- Iter 7 Costs = [26 35]
- Iter 8 Costs = [37 26]
- Iter 9 Costs = [26 35]
- Iter 10 Costs = [31.9 29.7]

FW

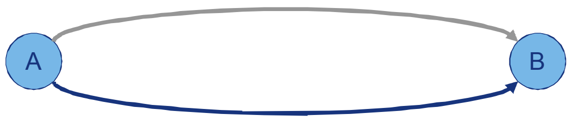
Demand A,B = 10



Begin by taking free-flow costs

FW – Iteration 1 – the same

$1 + x_1^2 = 1$



$10 + x_2^2 = 10$

So All-or-nothing flow assignment is

$x_1' = 10$ and $x_2' = 0$

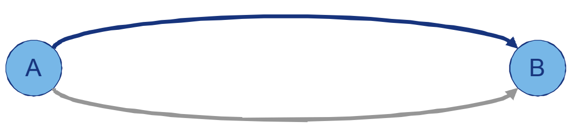
or $x = [\,10 \;\; 0\,]$

Calculate new costs based on these flows

FW - Iteration 2

$x = [\,10 \;\; 0\,]$

$1 + x_1^2 = 101$



$10 + x_2^2 = 10$

So All-or-nothing flow assignment is $x_1' = 0$ and $x_2' = 10$

or $x_i = (\lambda)\, x' + (1 - \lambda)\, x_{old}$ (the difference)

<u>But how to find λ?</u>

Calculate new costs based on these flows

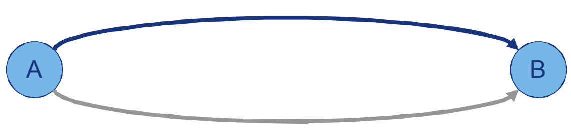
Finding λ

- Regardless of network size, number of zones, etc. there is a **single** variable λ
- Finding a single variable is relatively easy as long as we have an **objective f(λ)**
- For static that comes directly from the formulation we stated

FW - Iteration 2

$x = [\ 10 \ \ 0\ ]$

$1 + x_1^2 = 101$



$10 + x_2^2 = 10$

So All-or-nothing flow assignment is $x_1' = 0$ and $x_2' = 10$

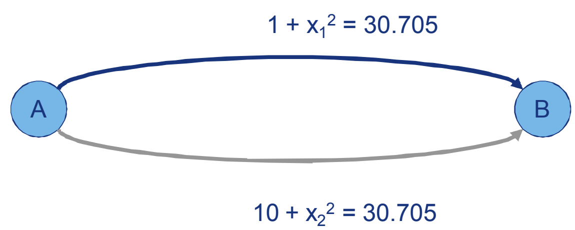
or $x_i = (\lambda)\, x' + (1 - \lambda)\, x_{old}$ (the difference)

λ can be found = .455
So $x = [\ 5.45 \ \ 4.55\ ]$

Calculate new costs based on these flows

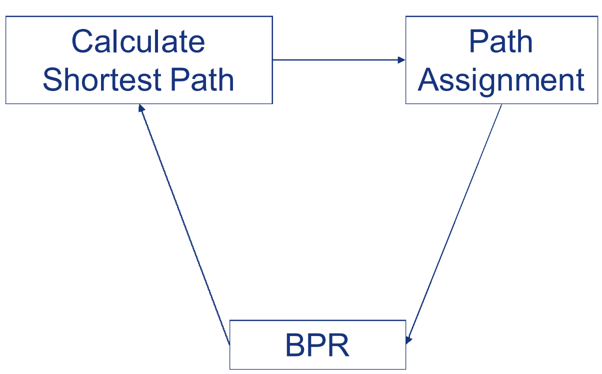
FW - Iteration 3

$x = [\ 5.45\ 4.55\ ]$



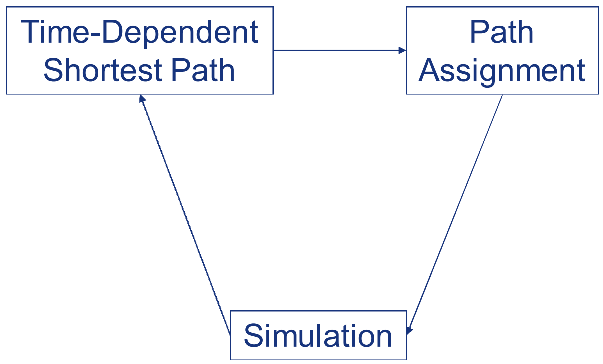
Stop

MSA/FW Static Assignment



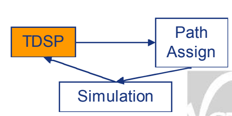
So, how do we move to DTA?

Simulation-based DTA



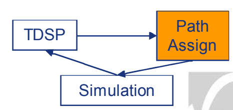
Simulation-based DTA: Time-Dependent Shortest Path

- Analogous to Shortest Path in static UE
  - Given an arrival time at destination (or departure time from origin), find shortest path
  - Link travel time depends on time of arrival
  - Relatively high fidelity may be needed to capture the impact of control, etc



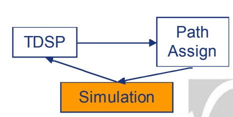
Simulation-based DTA: Path Assignment Component

- Multiple emerging DTA methods include:
- Method of Successive Averages
  - Uses fixed path splits over iterations
- Simplicial Decomposition
  - Employs objective functions
- Other “gap-based” methods



Supply: Desired Features of a Traffic Flow Model

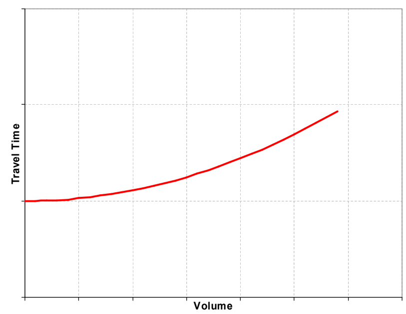
- Ability to model:
  - Bottlenecks
  - Link Spillover
  - Shockwave Propagation
- The correct modeling of these issues are critical to Dynamic Traffic Assignment



Modeling Congestion

A typical BPR volume-delay function:

$$t(v) = t_o \cdot \left(1 + \left(\frac{v}{c}\right)^{\alpha}\right)$$



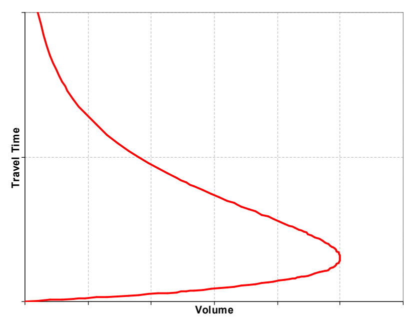
Modeling Congestion

Even simple traffic models provide a more accurate representation.

$$u(k) = u_f \cdot \left(1 - \frac{k}{k_j}\right)$$

$$q = u \cdot k$$

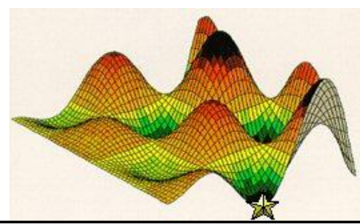
$$t(u) = \frac{t_0 \cdot u_f}{u}$$



Given the same flow, two travel times are possible. The difference is **density.**

Convergence

- Ideally, we would like to know:
  - “How far is the current solution from equilibrium?”
- Running “until things stop changing” may be problematic
  - Convergence of system cost may not imply convergence of link flow
  - Convergence of link flow may not imply equilibrium path costs



Simulation-based DTA: Path Assignment: Distance to Equilibrium

- DTA convergence offers opportunities
- An example of an equilibrium “gap” function
- Given an O-D with three used routes A, B and C:

Path	Flow	Travel Time	Least Travel Time	Gap
A	10	5	3	20
B	20	6	3	60
C	30	3	3	0
				80

Research Comparing DTA to Static

- It is surprisingly difficult to compare static and dynamic
  - If attempted incorrectly will greatly confound consistency of analysis
  - Another impact of V/C > 1 issues and such
- Also an issue because of boundary conditions/transient behavior
- If peak period analysis is attempted with DTA, a warm-up cool-down period is required
  - 24-hr modeling may be best approach

Sampling of Other Models

- The previous approach to DTA is essentially:
- **Equilibrium on Experienced Travel Cost**
- Numerous other models exist
  - Non-equilibrium approaches
  - "One-shot" Models
  - Instantaneous Travel time Models

Instantaneous Travel Time Versus Experienced Travel Time Modeling

Time	Link Travel Times		
0	1	2	3
1	2	3	4
2	1	4	4
3	1	3	5
4	2	3	5

- If departing at time 1:
- **Instantaneous travel time = 6** (just add up first row)
- Experienced travel time =

  Link 1 travel time (at time 0) = 1

  Link 2 travel time (at time 1) = 3

  Link 3 travel time (at time 4) = 5

  Experienced Travel Time = 9

One-shot Modeling

- “One-shot” models do not attempt an equilibrium in the previously described sense
- Traffic does spread over routes but not due to equilibration
  - One approach is to base route choice decisions at time T no congestion up to time T
  - For instance, use instantaneous travel times
- Most similar to incremental approaches for static assignment

Harsh Truth #1: Disequilibrium Versus Non-Convergence

- It is occasionally noted that traffic isn't really in equilibrium so why worry about it?
- However, if we want to use DTA for planning then we must have stable solutions
  - If noise or randomness substantially impact the solution, the results are not defensible in a planning context
  - Further, explainable behavior is important

Harsh Truth #1: Disequilibrium Versus Non-Convergence

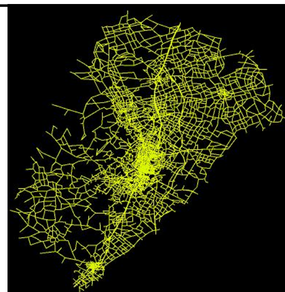
- There is ongoing research into "disequilibrium" (or transient) traffic modeling.
  - However, this research is clearly distinct from **non-convergence**
  - Put simply, stopping an equilibrium model prior to convergence is not correct (and is not supported by arguments related to traffic disequilibria)
  - For planning applications, any disequilibrium network model must still provide a stable solution (and current options for this appear limited)
  - Equilibrium remains the simplest approach to generate stable solutions for planning applications

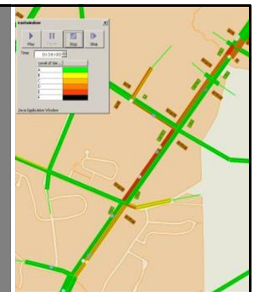
Harsh Truth #2: Costs of DTA

- If a DTA approach does not result in substantially higher computational costs, there are two highly likely reasons:
  - Very few (if any) behavioral advantages are being obtained
  - Substantial sacrifices are being made in terms of solution quality, convergence, stability, etc
- However, many questions can only be answered given the superior behavioral representation of DTA

Conclusions

- A wide variety of models may currently be termed DTA
- Without any modifying term, the Primer defines DTA as an **Equilibrium Based on Experienced Travel Cost**
- Equilibrium remains the simplest approach to obtain stable solutions for planning applications
- Convergence and stability are still absolutely critical for planning applications (using DTA should not change this)
- As a new consideration for planners, traffic realism is also critical





THANK YOU!

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